Part 1

Chapter13

Introduction to Acceptance Sampling

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13.1 What It Means

This was popularized by Dodge and Romig, and was originally applied by the U.S. military to the testing of bullets during World War II.

This process is called *Lot Acceptance Sampling* or just *Acceptance Sampling*.

Deciding about the lot, based on what has been found in sample is, in statistics, known as Statistical Inference.

Two common types of acceptance sampling are:

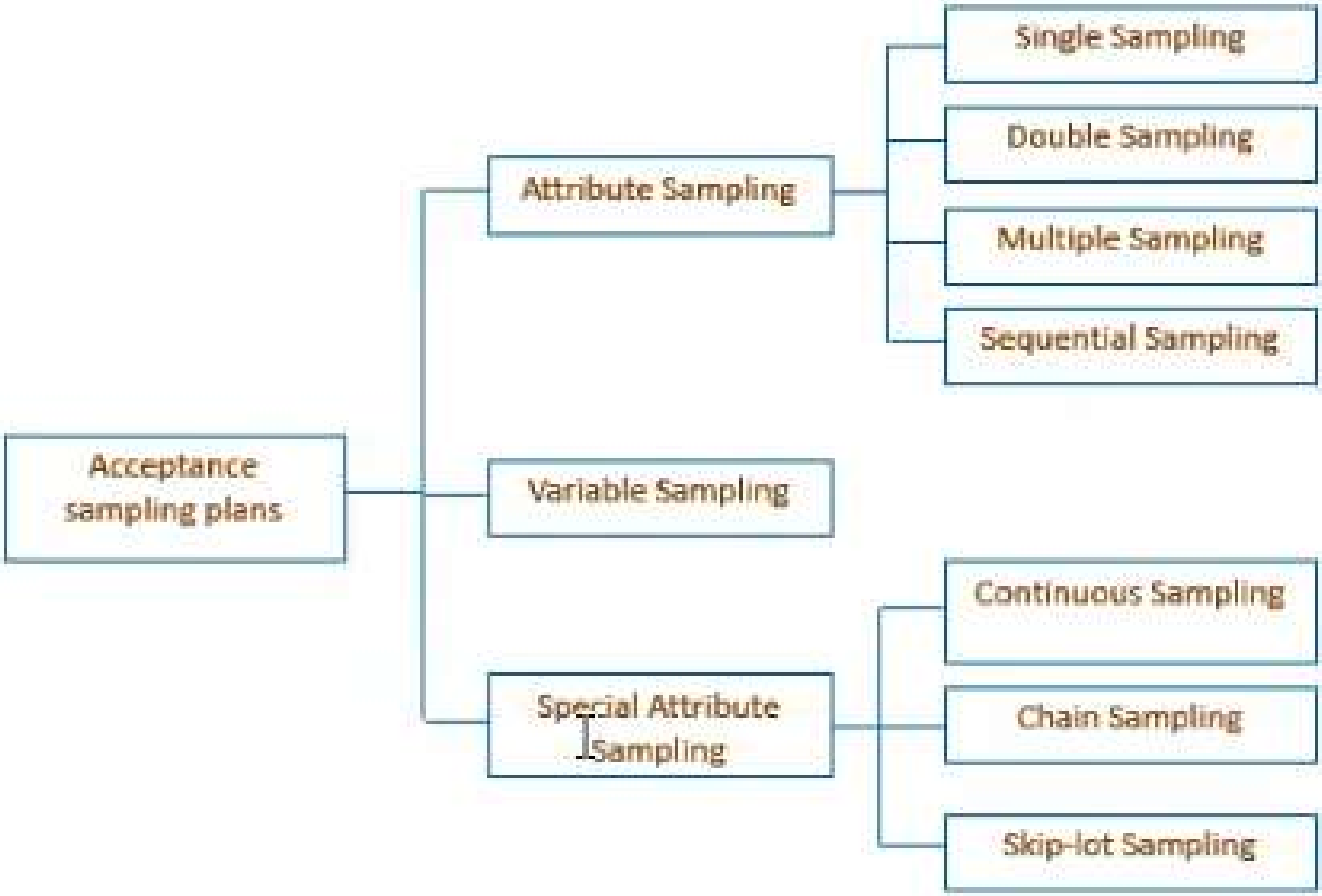
1. Acceptance Sampling by *Attributes* ("go, no-go", or similar to control chart for attributes)
2. Acceptance Sampling by *Variables* (similar to control chart for variables).

There are several situations when 100% inspection is not often practical:

- When testing is destructive, otherwise all the products will be lost
- When inspection costs are very high
- When many similar products are to be inspected
- When efforts required for testing is very high
- When time and technology limitations are high
- When the population or lot size is very large
- When the population is geographically scattered over a large area
- Supplier's quality history is good enough to justify less than 100% inspection

Sampling gives you a reasonable solution to this problem

13.3 Types of Sampling plans



13.4 Some Definitions Related to Sampling Plans

Acceptable Quality Level (AQL):

This is the poorest quality level of the supplier's (or the producer's) process that the consumer would consider to be acceptable as a process average.

This can also be considered as a percent defective that is the base line requirement for the quality of the producer's product.

13.5 Parameters and Symbols Used

N = Total Number of products/units in a lot or batch
n = Number of products/units to be inspected from a batch
D = Number of nonconforming products/units in a sample of size n
c = Acceptance number, or maximum allowable number of nonconforming products in a sample of size n
p = Fraction nonconforming
P_a = Probability of acceptance
α = Producer's risk (probability of Type I error, as defined earlier)
β = Consumer's risk (probability of Type II error, as defined earlier)

13.7 Single sampling plan

Single sampling plan is the simplest and the shortest plan.

A sample (of size *n*) is drawn from a batch or lot (of size *N*).

If the number of nonconforming units is less than a predetermined number (known as acceptance number c), then the lot or batch is accepted, otherwise rejected.

Let's assume that a sampling plan states that –

Lot size $N = 1000$,   sample size $n = 30$ and acceptance number $c = 3$.

If no. of nonconforming units ≤ 3 pieces → Accept the entire lot

If no. of nonconforming units > 3 pieces → Reject the entire lot

Since the acceptance ('go, no-go' type) follows ***binomial distribution***, the probability of acceptance has the following equation.

$$P_a = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$ *where, p* is the fraction nonconforming ....(1)

Example: Single Sampling Plan

A company produces a batch of 1000 products.

An agreement between the producer and the customer specifies the followings:

Order Batch size N = 1000 units, Sample size n = 30 units, Acceptance number c = 2

Plot the OC curve for this sampling plan.

Solution:

P_a = Probability of acceptance ; p = Fraction nonconforming.

For p =0.01 ➡

$$P_a = \sum_{i=0}^{2} \binom{30}{i} (0.01)^i (1-0.01)^{30-i}$$

= P (0 nonconforming) + P (1 nonconforming) + P (2 nonconforming)

$$= \left[\binom{30}{0}(0.01)^0(1-0.01)^{30-0}\right] + \left[\binom{30}{1}(0.01)^1(1-0.01)^{30-1}\right] + \left[\binom{30}{2}(0.01)^2(1-0.01)^{30-2}\right]$$

= [0.7397] + [0.22415] + [0.03283] = **0.99668**

Similarly,

For $p = 0.03 \rightarrow P_a = [0.4010] + [0.37206] + [0.16685] = 0.93991$

For $p = 0.05 \rightarrow P_a = [0.214638] + [0.3389] + [0.258636] = 0.81217$

For $p = 0.07 \rightarrow P_a = [0.11336] + [0.25599] + [0.27938] = 0.64873$

For $p = 0.09 \rightarrow P_a = [0.05905] + [0.1752] + [0.25126] = 0.48551$

For $p = 0.11 \rightarrow P_a = [0.030317] + [0.112414] + [0.20146] = 0.34419$

For $p = 0.15 \rightarrow P_a = [0.00763] + [0.04039] + [0.10337] = 0.15139$

For $p = 0.17 \rightarrow P_a = [0.003735] + [0.02295] + [0.068166] = 0.09485$

Table 13.1: Probability of acceptance values.

Fraction nonconforming, p	Probability of acceptance, P_a
0.01	0.99668
0.03	0.93991
0.05	0.81217
0.07	0.64873
0.09	0.48551
0.11	0.34419
0.15	0.15139
0.17	0.09485

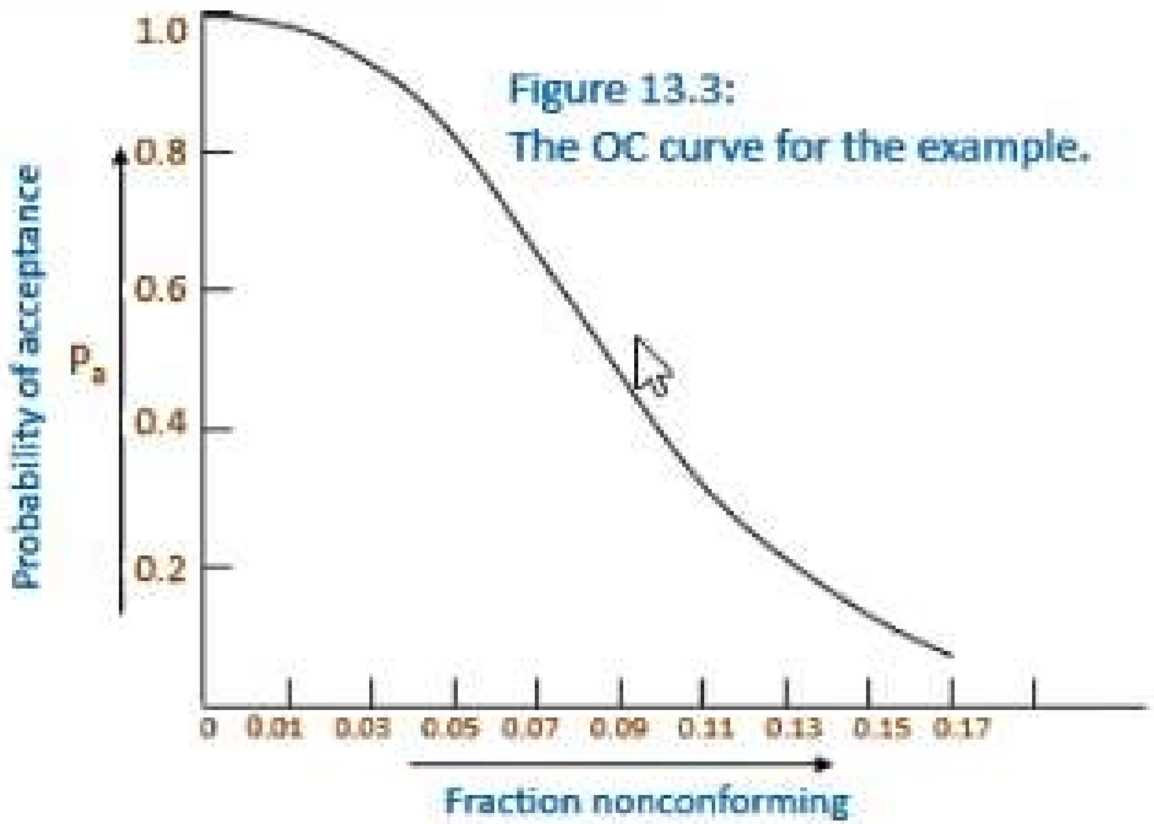
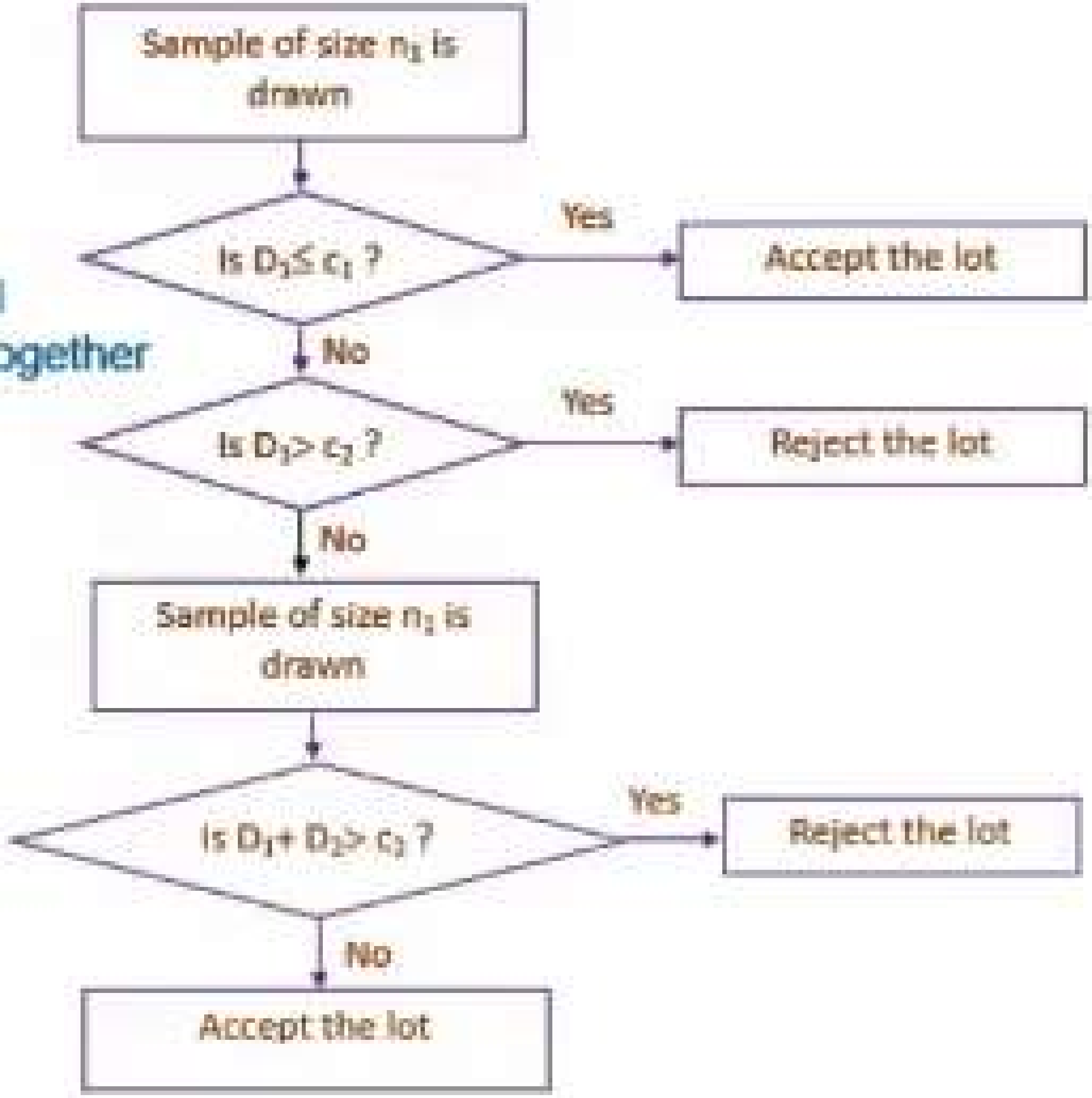


Figure 13.3:
The OC curve for the example.

13.8 Double Sampling Plans

n_1 = Sample size in the first trial
n_2 = Sample size in the second trial
c_1 = Acceptance number for the first trial
c_2 = Acceptance number for both trials together
D_1 = Number of nonconforming units
D_2 = Number of nonconforming units



The above procedure can be explained with an example. Suppose that –

$N = 3000$, $n_1 = 40$, $n_2 = 80$, $c_1 = 1$, $c_2 = 4$

- The above double sampling plan parameters mean that a batch of products containing 3000 units is to be inspected.

- In the first attempt (trial), 40 units are randomly drawn and inspected. If the number of nonconforming units $D_1 \leq 1$, then the entire lot (3000 units) is accepted, without requiring the second trial.

- If the number of nonconforming units $D_1 > 4$, then the entire lot is rejected, without requiring the second trial.

- If the number of nonconforming units $D_1 = 2$, or 3 or 4, then another sample of size 80 units is taken and inspected.

- If the number of nonconforming units from both the samples $(D_1 + D_2) \leq 4$, then the lot is accepted. Otherwise, the lot is rejected.

Example: Double Sampling Plan

A company and its customer have agreed to follow a double sampling plan, with the following parameters:

Lot size $N = 3000$, First sample size $n_1 = 40$, $c_1 = 2$, Second sample size $n_2 = 80$, $c_2 = 4$,

P_a^1 = Probability of acceptance in the first sample; P_a^{11} = Probability of acceptance in the second sample

P_a = Total probability of acceptance in the combined first and second sample $= P_a^1 + P_a^{11}$

The lot is accepted after the first sample, if $D_1 = 0$, or 1, or 2. Thus, for fraction nonconforming value, $p = 0.05$, probability of acceptance in the first sample is -

$$P_a^1 = \sum_{i=0}^{2} \binom{40}{i} (0.05)^i (1-0.05)^{40-i}$$

= P (0 nonconforming) + P (1 nonconforming) + P (2 nonconforming)

$$= \left[\binom{40}{0}(0.05)^0(1-0.05)^{40-0}\right] + \left[\binom{40}{1}(0.05)^1(1-0.05)^{40-1}\right] + \left[\binom{40}{2}(0.05)^2(1-0.05)^{40-2}\right]$$

= [0.128512] + [0.27055] + [0.27767]

= 0.67673 ← probability of acceptance in the first trial ....(1)

If $D_1 > 4$, then the entire lot is rejected, without taking the second sample.

A second sample (of size 80) is taken only if $D_1 = 3$ or 4.

The lot will be acceptable after 2nd trial, if $(D_1 + D_2) = 3$ or 4.

Probable combinations for this situation are given below:

$D_1 = 3$ and $D_2 = 0$ → Probability of acceptance is $P_a^{11}(D_1 = 3, D_2 = 0)$

or

$D_1 = 3$ and $D_2 = 1$ → Probability of acceptance is $P_a^{11}(D_1 = 3, D_2 = 1)$

or

$D_1 = 4$ and $D_2 = 0$ → Probability of acceptance is $P_a^{11}(D_1 = 4, D_2 = 0)$

$$P_a^{11}(D_1 = 3, D_2 = 0) = \left[\binom{40}{3}(0.05)^3(1-0.05)^{40-3}\right] \times \left[\binom{80}{0}(0.05)^0(1-0.05)^{80-0}\right] = 0.003057$$

$$P_a^{11}(D_1 = 3, D_2 = 1) = \left[\binom{40}{3}(0.05)^3(1-0.05)^{40-3}\right] \times \left[\binom{80}{1}(0.05)^1(1-0.05)^{80-1}\right] = 0.01287$$

$$P_a^{11}(D_1 = 4, D_2 = 0) = \left[\binom{40}{4}(0.05)^4(1-0.05)^{40-4}\right] \times \left[\binom{80}{0}(0.05)^0(1-0.05)^{80-0}\right] = 0.001488$$

Thus, for the above three combinations, total probability of acceptance in 2nd trial is -

$P_a^{11} = 0.003057 + .01287 + 0.001488$

$= 0.017415$ ← Prob. of acceptance in the 2nd trial ....(2)

As a result, total probability of acceptance in two trials is (from eq. 1and 2)–

$P_a = P_a^1 + P_a^{11}$ = 0.67673 + 0.017415 = 0.694145 ....for $p = 0.05$;

Similarly for other p values, probability of acceptance need to e calculated, for construction of OC curve.

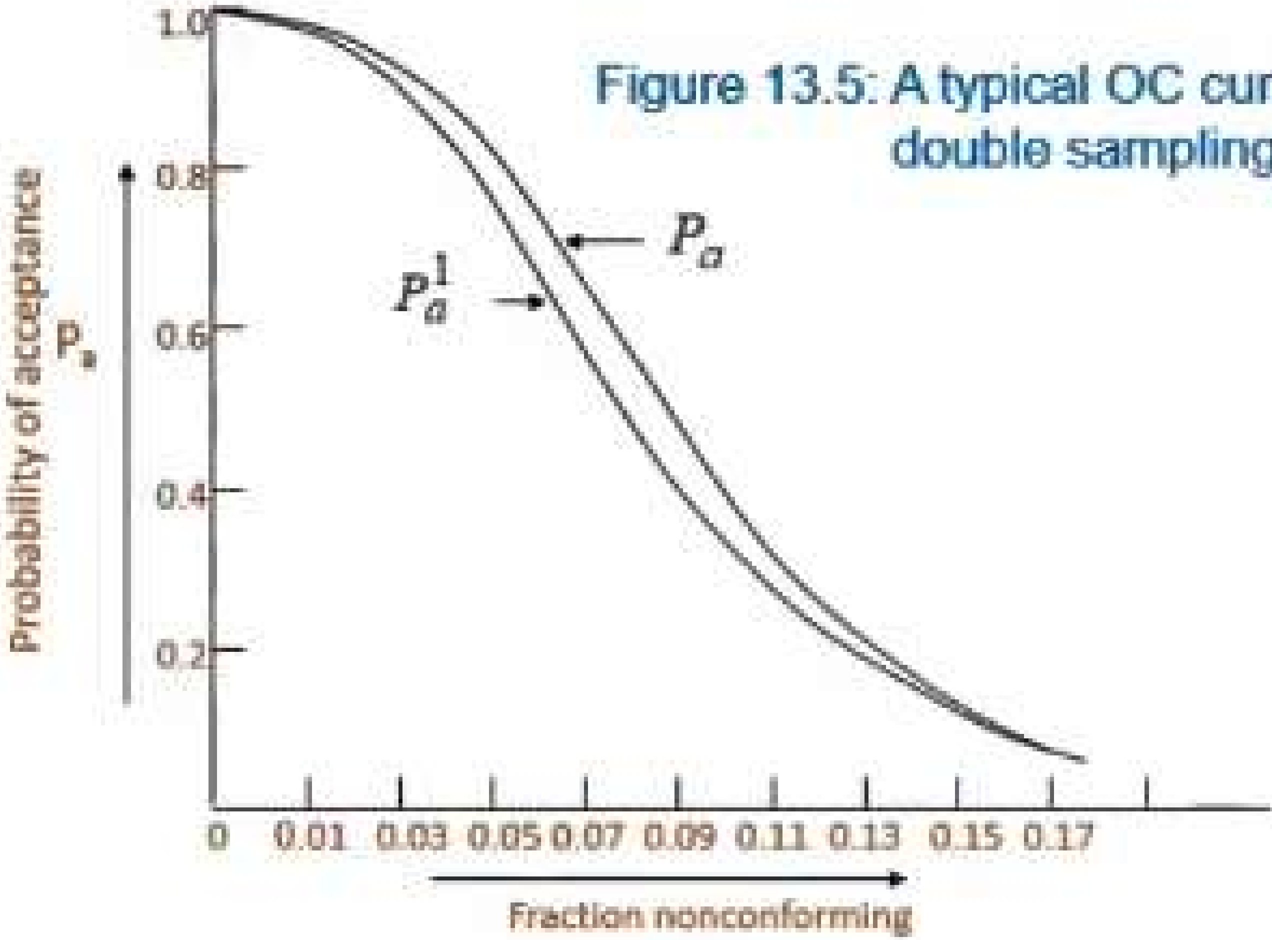
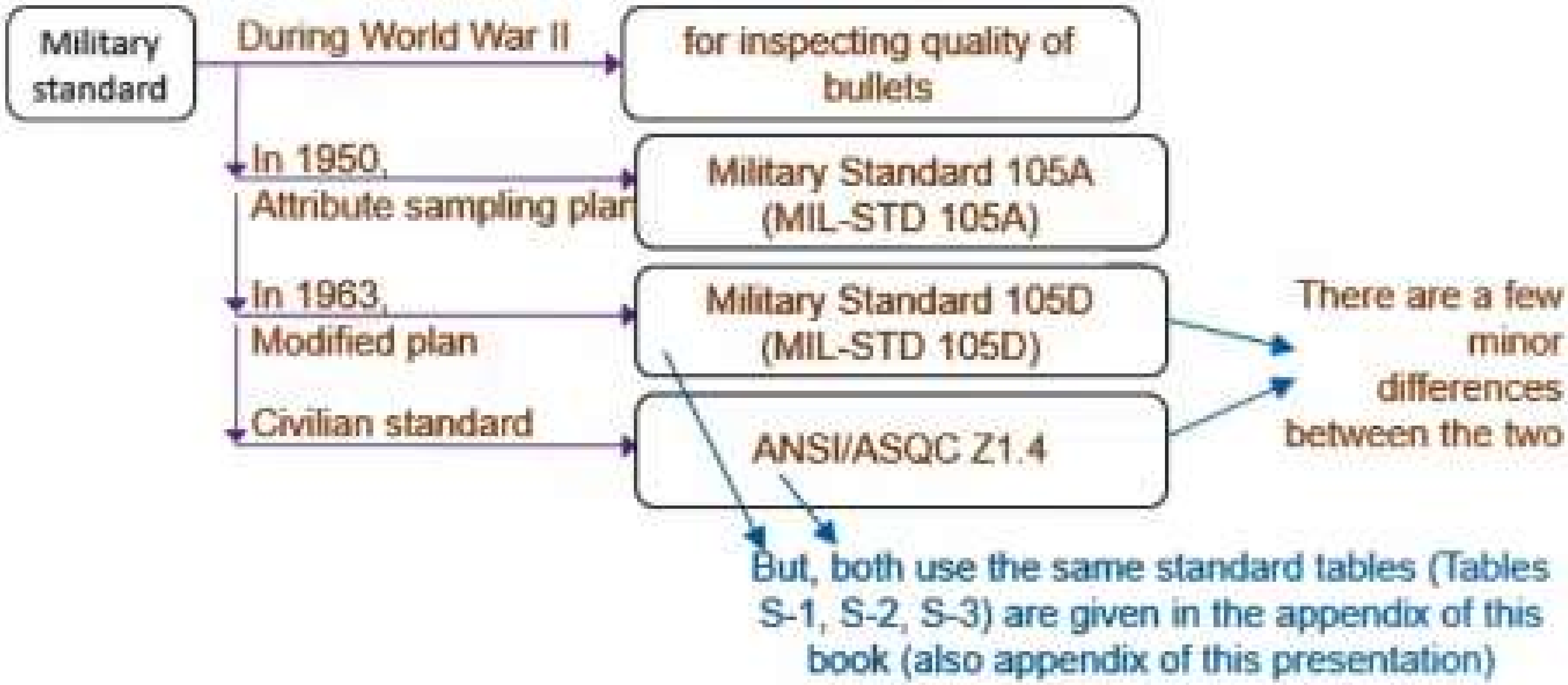


Figure 13.5: A typical OC curve for a double sampling plan

13.11 Military and ANSI Standards



Since a large number of buyers of Ready-made Garments are from the USA, many RMG producers in Bangladesh are enforced by the buyers to follow such standards.

This standard additionally specifies a set of rules to switch from one type of inspection to another (e.g. normal, tightened, and reduced inspection) and inspection levels (e.g. Level I, II and III). These are explained in the following sections.

13.11.1 Type of inspection and rule of switching

Three types of inspections are available, depending upon degree of severity of nonconforming situation, or quality status of products:

1) Normal inspection
2) Tightened inspection
3) Reduced inspection

The quality inspection starts with *Normal Inspection* and continues as long as the products are at AQL or better.

Figure 13.9: Switching between rules.

Switching rules

1. Normal to Tightened

As stated earlier, the process starts with Normal Inspection. One can switch to Tightened Inspection if two out of five consecutive lots/batches are rejected. The idea behind this switching is to impose tight control on inspection.

2. Tightened to Normal

When Tightened Inspection is active, one can switch to Normal Inspection if five consecutive lots/batches are acceptable.

3. Normal to Reduced

When Normal Inspection is active, one can switch to Reduced Inspection when all four conditions, stated below, are true:

a) The earlier 10 consecutive lots are acceptable under Normal Inspection.
b) Total number of nonconforming units in the earlier 10 lots is equal to or less than the acceptance limit numbers.
c) Production is steady and no signal is evident for any untoward incidence.
d) Reduced Inspection is desired for some meaningful purpose.

Similarly "Reduced to Normal"……

13.11.2 Inspection level

The sample size is determined based on – batch/lot size and inspection level required.

There are three inspection levels in practice – Level I, II and III.

1) The second level (level II) is practiced under normal situation.
2) 1st level (Level I): for loose control with less accuracy; requires about half of the sample size taken in Level II.
3) 3rd level (Level III): for tight control with high accuracy; requires about double of the sample size taken in Level II.

Table S-1: Sampling Plan Table (MIL-STD 105D and ANSI/ASQC Z1.4)

Lot size (units)	General Inspection Levels		
	I	II	III
2 – 8	A	A	B
9 – 15	A	B	C
16 – 25	B	C	D
26 – 50	C	D	E
51 – 90	C	E	F
91 – 150	D	F	G
151 – 280	E	G	H
281 – 500	F	H	J
501 – 1200	G	J	K
1201 – 3200	H	**K**	L
3201 – 10,000	J	L	M
10,001 – 35,000	K	M	N
35,001 – 150,000	L	N	P
150,001 – 500,000	M	P	Q
500,001 – more	N	Q	R

N.B. Selection of an Inspection Level depends on production system type and accuracy required (Normal : Level 2).

Table S-2: Single Sampling Plan (MIL-STD 105D and ANSI/ASQC Z1.4)

Sample Size code	Sample size	Acceptable Quality Level (AQL) 1% A	1% R	1.5% A	1.5% R	2.5% A	2.5% R	4.0% A	4.0% R	6.5% A	6.5% R	10% A	10% R	15% A	15% R
A	2									0	1				
B	3							0	1					1	2
C	5					0	1					1	2	2	3
D	8			0	1					1	2	2	3	3	4
E	13	0	1					1	2	2	3	3	4	5	6
F	20					1	2	2	3	3	4	5	6	7	8
G	32			1	2	2	3	3	4	5	6	7	8	10	11
H	50	1	2	2	3	3	4	5	6	7	8	10	11	14	15
J	80	2	3	3	4	5	6	7	8	10	11	14	15	21	22
K	**125**	3	4	5	6	7	8	**10**	**11**	14	15	21	22		
L	200	5	6	7	8	10	11	14	15	21	22				
M	315	7	8	10	11	14	15	21	22						
N	500	10	11	14	15	21	22								
P	800	14	15	21	22										
Q	1250	21	22												
R	2000														

A = Acceptable number of defects
R = Rejectable (Rejection) number of defects

Note: Similarly, there is "Double sampling" table S-3